



## PhD Proposal 2017

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<b>Collaboration with other partner during this PhD:</b> <b>In France:</b>	<b>In China:</b>

<b>Title: Numerical optimization within MCMC for statistical inference in high dimensions</b>
<b>Scientific field (*): <i>Computer Science, Image and data processing, applied mathematics</i></b>
<b>Key words: image restoration, spectral imaging, inverse problems, Bayesian inference, Monte Carlo methods, numerical optimization</b>

## **Details for the subject:**

### **Numerical optimization within MCMC for statistical inference in high dimensions**

#### **Background, Context:**

This thesis project is part of research activities of the group of signal and image processing at Ecole central de Nantes at IRCCyN Laboratory (Institut de Recherche en Communications et Cybernétique de Nantes), which is related to the development of algorithmic tools for the resolution of large scale inverse problems.

#### **Research subject, work plan:**

The resolution of several statistical inference problems in signal and image processing is performed using stochastic simulation tools or numerical optimization methods. For instance, for Bayesian image restoration or reconstruction, the estimated image is calculated from the posterior distribution using either the minimum mean square error estimator, which is generally obtained by Markov chain Monte Carlo methods, or the maximum a posterior estimator, which is calculated using an iterative descent method. Actually, stochastic simulation methods offers the advantages of being able to jointly estimate all the parameters for the posterior distribution using a hierarchical Bayesian model. Such methods are therefore well adapted to introduce more elaborate prior models on the unknown variables. However, their applicability is limited by the excessive computational burden when dealing with high dimensional inference problems (problems with high number of unknowns or variables).

The main objective of this thesis is to develop stochastic simulation strategies incorporating numerical optimization methods in order to deal with high dimensional inverse problems, by either reducing the computation cost or accelerating the algorithm convergence. A typical example is the sampling of a multivariate Gaussian distribution, whose subset of variables exhibit high mutual correlations. Several works have shown that introducing some dynamics inspired from the optimization theory (Hamiltonian Monte Carlo, Langevin-Hastings) allows to built Markov chains whose convergence rate is enhanced. However, the analysis of the convergence of these algorithms and their variants remains an active research area [Aune, 2013].

We recently proposed in [Gilavert, 2015] a method for sampling multivariate Gaussian distributions in high dimension. This method incorporates some dynamics guided by an approximate resolution of a linear system. The first part of this thesis will be dedicated to the exploration of the different possibles extensions of this method, its link with existing approaches and extension to the non Gaussian case. The second part of the thesis will be focused on the optimisation of the computation cost of hybrid Monte carlo methods using an adaptive tuning of setup parameters [Roberts, 1998]. An expected original result will be the optimal tuning of Langevin and Hamiltonian methods in terms of statistical efficiency measured by the computation cost per effective sample.

Finally, the proposed methods will be applied to real applications related ti signal and image restoration in physical problems.

## References

[Gilavert, 2015] C. Gilavert, S. Moussaoui et J. Idier, Efficient Gaussian sampling for solving large-scale inverse problems using MCMC, *IEEE Trans. Signal Processing*, vol. 63, no. 1, pp.70-80, 2015.

[Aune, 2013] E. Aune, J. Eidsvik, and Y. Pokern, Iterative numerical methods for sampling from high dimensional Gaussian distributions, *Statist. Comp.*, pp. 1–21, 2013.

[Roberts, 1998] G. O. Roberts and J. S. Rosenthal, “Optimal scaling of discrete approximations to Langevin’s diffusions,” *J. R. Statist. Soc. B*, vol. 60, no. 1, pp. 255–268, 1998