PhD Proposal 2017

School: Ecole Centrale de Nantes	
Laboratory: GeM	**Web site: https://gem.ec-nantes.fr/**
Teams : Modeling and Simulation – Materials, Environment, Structures.	**Head of the team:**
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Collaboration with other partner during this PhD: **In France:**	**In China:**

Title: Multi-scale modeling of periodic heterogeneous media under complex loading conditions.	
Scientific field: Continuum mechanics, Applied Mathematics	
Key words: Heterogeneous media, Gradient theories, asymptotic expansions	

Details for the subject:

Background, Context:

Modeling the behavior of heterogeneous structures, under complex loading conditions, typically requires the characterization of non-classical phenomena : the so-called size-effect, which have to be incorporated into the structural behavior to accurately reproduce its overall response . Examples of such kind of behavior are ubiquitous in natural and artificial materials and structures. One can refer for instance to the behavior of micro porous or granular materials, as concrete, clays or shales, but also to honeycomb micro-structures, artificially built up from a 3D printer. Intrinsic characteristic lengths of the material are in these cases expected to directly influence the overall constitutive law of the heterogeneous medium, or in other words the constitutive law will not more be that of a simple material but non local effects will be of main relevance.

A typical procedure to account for these effects at the macroscopic scale is that of considering enhanced modeling schemes, in the framework of continuum mechanics, which means modeling materials as micro-structured or gradient continua. In this case the effects of intrinsic characteristic lengths is naturally introduced into the continuum description, enriching the kinematical description (micro polar and Cosserat continua) or regarding the stored energy as a function not only of strain but also of strain gradients (gradient theories). In this project we shall focus our attention on this last class of continua, in particular on strain gradient or second gradient continua, which even in the regime of elasticity provide a natural way to import non local effects into the model.

Apparently the main question concerning the physical interpretation of such kind of models regards how the intrinsic characteristic lengths, or analogously the corresponding constitutive parameter, which provide the envisaged enhanced feature of the model, can be estimated. Indeed a natural procedure which allows to determine these new parameters is the asymptotic expansion method which is a rigorous tool to derive the overall response of periodic elastic structures. Assuming the material to be periodic at the microscopic scale, which could be a reasonable mathematical schematization of the intrinsic micro-structure of natural materials, or even a specific assumption in the design of artificial micro-structures, and assuming, at this level, a classical continuum description of its response, a cascade of problem, and equations, is stated once the unknown (displacement) field is developed, with respect to a small parameter ε, in terms of a double scale expansion:

$$u_\varepsilon(x) = u_0\left(x, \frac{x}{\varepsilon}\right) + \varepsilon u_1\left(x, \frac{x}{\varepsilon}\right) + \varepsilon^2 u_2\left(x, \frac{x}{\varepsilon}\right) + \varepsilon^3 u_3\left(x, \frac{x}{\varepsilon}\right) + \dots$$

The problem, originally formulated at the microscopic scale, is therefore decomposed into a cascade of micro-scale heterogeneous problems, posed on a small domain: the basic cell (the period) of the structure, and parametrized by the macroscopic displacement, strain and strain gradient. Using suitable compatibility conditions for the micro-scale problems the proper macroscopic scale equations can be deduced, starting from the classical one of continuum mechanics up to that of strain gradient continua, so providing a characterization of the enhanced constitutive laws. This macroscopic problem is therefore homogeneous and thus can be solved using a finite element model with a coarse mesh.

Such kind of approach has been discussed in [1-4] for periodic media, or in [5] for slender periodic beamlike structures with an overall 1D behavior, but the application of the method to practical examples, especially by means of numerical methods, appear to be limited.

The subject of the PhD thesis will concern the construction of the abovementioned cascade of micro-scale problems for specific elastic microstructures. The numerical solution of the sequence of the micro-scale problems will be obtained implementing a suitable finite element code, and consequently the macro-scale constitutive equations will be deduced.
The non-local constitutive law relative to the enhanced macro-scale gradient model will be implemented within a finite element code in order to represent specific features of the heterogeneous material response to complex loading conditions.

One possible application will concern fracture mechanics and in particular the characterization of stress, generalized stress and strain in the neighborhood of the crack tip.

The accuracy of the multi-scale model will be assessed by comparison with :

- the 3D heterogeneous solution computed from a detailed finite element model of the heterogeneous structure;
- experimental results obtained from full-field measurements using digital image correlation.

A background in continuum mechanics is highly recommended, together with non elementary skills in computer programming.

Research subject, work plan:
The research mainly concerns a rigorous approach to modeling the response of heterogeneous materials to complex loadings.
A work planning will be established at the beginning of the three years in order to provide a clear temporal scheduling of the activities of the student. In principle the first year will be devoted to the understanding and statement of the problem, the second year and a half to the numerical implementation of the finite element codes at the micro and the macroscopic scales. The residual half year will be devoted to the redaction of the thesis.

References:

[1] Boutin C. (1996) Microstructural effects in elastic composites. International Journal of Solids and Structures 33(7), 1023-1051

[2] Smyshlyaev, V. P., & Cherednichenko, K. D. (2000). On rigorous derivation of strain gradient effects in the overall behaviour of periodic heterogeneous media.Journal of the Mechanics and Physics of Solids, 48(6), 1325-1357.

[3] Tran, T. H., Monchiet, V., & Bonnet, G. (2012). A micromechanics-based approach for the derivation of constitutive elastic coefficients of strain-gradient media. International Journal of Solids and Structures, 49(5), 783-792.

[4] Li, J., & Zhang, X. B. (2013). A numerical approach for the establishment of strain gradient constitutive relations in periodic heterogeneous materials.European Journal of Mechanics-A/Solids, 41, 70-85.

[5] Buannic, N., & Cartraud, P. (2001). Higher-order effective modeling of periodic heterogeneous beams. I. Asymptotic expansion method. International Journal of Solids and Structures, 38(40), 7139-7161.